Evidently a monoclinic system material is determined by 12 independent parameters: d_1, $d_2, \ldots$, d_6, c_{21}, c_{31}, c_{41}, c_{32}, c_{42}, and c_{43} with condition (1.5) being satisfied. If the moduli E_2, E_3, and $E_4 = 2\mu_{23}$ are taken as the independent parameters instead of d_2, d_3, and d_4, then by using (1.5) and the technical notation (1.8), we obtain condition (2.9) and

$$\frac{1}{E_4} > \frac{\nu_{41}^2}{E_1} + \frac{(\nu_{42}/E_2 - \nu_{41}\nu_{21}/E_1)^2}{1/E_2 - \nu_{21}^2/E_1} + \frac{\left[\frac{\nu_{43}}{E_3} - \frac{\nu_{41}\nu_{31}}{E_1} - \frac{(\nu_{42}/E_2 - \nu_{41}\nu_{21}/E_1)(\nu_{32}/E_2 - \nu_{31}\nu_{21}/E_1)}{1/E_2 - \nu_{21}^2/E_1}\right]^2}{\frac{1}{E_3} - \frac{\nu_{31}^2}{E_1} - \frac{(\nu_{32}/E_2 - \nu_{31}\nu_{21}/E_1)^2}{1/E_2 - \nu_{21}^2/E_1}}. \tag{2.10}$$

For <u>triclinic system</u> materials, the matrices a_{ij} and A_{ij} have the general form and therefore the formulas (1.3)-(1.7), (1.10)-(1.15) for the general anisotropic case must be used. The conditions $d_5 > 0$, $d_6 > 0$ can be rewritten similarly to (2.9) and (2.10) by using the technical notation (1.8), but due to the unwieldiness of these formulas, they will not be written out here.

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ENERGY VERSION OF CREEP AND STRESS-RUPTURE STRENGTH THEORY FOR ANISOTROPIC AND ISOTROPIC MATERIALS WHICH DIFFER IN RESISTANCE TO TENSION AND COMPRESSION

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UDC 539.3

A new separate branch in solid mechanics has recently been formed, i.e., creep theory for materials which resist tension and compression differently [1-15]. Intense development of it is connected with considerable engineering applications since light alloys, gray cast irons, polymers, ceramics, composites, and other materials whose creep depends on the type of loading are used extensively in various fields of technology. On the other hand, in published works [16-26] considerable attention is devoted to the mechanics of damaged materials. The majority of the approaches in this field are development and generalization of the Rabotnov concept [27] about a material damage parameter. It is evident that deformation and damage accumulation occur under creep conditions in parallel with each other and they have a reciprocal effect. In order to describe these phenomena it is very convenient to use equations of state in an energy form which make it possible to compare creep analysis with finding the time for failure of a structure. Here in the equations it is necessary to reflect the effect of the form of loading on creep and stress-rupture strength.

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1. In order to construct a connection between components of symmetrical tensors for creep strain rate $\dot{\varepsilon}$ and stresses σ (second rank tensors) in anisotropic materials we use a potential

$$F = \sigma_e^2. \tag{1.1}$$

Then

$$\dot{\boldsymbol{\varepsilon}} = \lambda \partial F / \partial \boldsymbol{\sigma}. \tag{1.2}$$

Here σ_e is equivalent stress $(\sigma_e \geqslant 0)$; λ is a scalar multiple subject to determination; the dot means derivative with respect to time t; strains are assumed to be small.

The equivalent stress, which establishes equivalence of uniaxial and complex stressed states, should be a uniform function of stress tensor invariants and certain constants. The physical state of the anisotropic material in question is described by the tensors of constants $\mathbf{b}$, ${}^{(4)}\mathbf{a}$, and ${}^{(6)}\mathbf{c}$ of second, fourth, and sixth ranks, respectively. Then we form mixed invariants of stress tensors and constants: linear $\sigma_1 = \mathbf{b} \cdot\cdot\, \boldsymbol{\sigma}$, quadratic $\sigma_0^2 = \boldsymbol{\sigma} \cdot\cdot\, {}^{(4)}\mathbf{a} \cdot\cdot\, \boldsymbol{\sigma}$, and cubic $\sigma_2^3 = \boldsymbol{\sigma} \cdot\cdot\, (\boldsymbol{\sigma} \cdot\cdot\, {}^{(6)}\mathbf{c} \cdot\cdot\, \boldsymbol{\sigma})$ (dots indicate summation with respect to indices repeated in tensors which take the values 1, 2, and 3). Then we write an expression for equivalent stress

$$\sigma_e = \sigma_0 + \alpha\sigma_1 + \gamma\sigma_2, \tag{1.3}$$

where α and γ are numerical coefficients which take account of the specific weight for different uneven combined invariants in the representation for σ_e.

Expression (1.3) is quite general and it includes a number of particular cases. For example, by placing $\alpha = \gamma = 0$ in (1.3) we obtain the relationship $\sigma_e = \sigma_0$ used for traditional anisotropoic materials [20, 27]. If we take $\alpha = 1$, $\gamma = 0$ in (1.3) we arrive at an expression for equivalent stress $\sigma_e = \sigma_0 + \sigma_1$ suggested previously in [12].

Then by differentiating (1.2) using (1.1) and (1.3) we have

$$\dot{\varepsilon} = 2\lambda\sigma_e(\partial\sigma_0/\partial\boldsymbol{\sigma} + \alpha\partial\sigma_1/\partial\boldsymbol{\sigma} + \gamma\partial\sigma_2/\partial\boldsymbol{\sigma}). \tag{1.4}$$

Taking account of the equations

$$\partial\sigma_0/\partial\boldsymbol{\sigma} = {}^{(4)}\mathbf{a} \cdot\cdot\, \boldsymbol{\sigma}/\sigma_0, \quad \partial\sigma_1/\partial\boldsymbol{\sigma} = \mathbf{b}, \quad \partial\sigma_2/\partial\boldsymbol{\sigma} = \boldsymbol{\sigma} \cdot\cdot\, {}^{(6)}\mathbf{c} \cdot\cdot\, \boldsymbol{\sigma}/\sigma_2^2,$$

we arrive from (1.4) at the equations

$$\dot{\boldsymbol{\varepsilon}} = 2\lambda\sigma_e\left[{}^{(4)}\mathbf{a} \cdot\cdot\, \boldsymbol{\sigma}/\sigma_0 + \alpha\mathbf{b} + \gamma\left(\boldsymbol{\sigma} \cdot\cdot\, {}^{(6)}\mathbf{c} \cdot\cdot\, \boldsymbol{\sigma}/\sigma_2^2\right)\right]. \tag{1.5}$$

Then by multiplying the right- and left-hand parts of (1.5) by σ and summing we obtain for specific dissipating capacity $W = \boldsymbol{\sigma} \cdot\cdot\, \dot{\boldsymbol{\varepsilon}}$ a relationship $W = 2\lambda\sigma_e^2$, i.e., in relationships (1.5)

$$2\lambda\sigma_e = W/\sigma_e. \tag{1.6}$$

As a measure of intensity of the creep process we take the specific dissipating capacity W, and as a measure of material damage we take the specific dissipation energy $\varphi = \int W dt$. We assume that independent of the form of stressed state the amount of energy dissipated at the instant of failure with creep $\varphi_* = \text{const}$. We take the following equation of state which connects at a fixed temperature processes of creep and failure:

$$W = f(\sigma_e, \varphi). \tag{1.7}$$

One of the forms of entry (1.7) for a disordered material may be

$$W = \frac{v(\sigma_e)\,\varphi_*^q}{(\varphi_* - \varphi)^q} \tag{1.8}$$

(q is a constant). We note that for a strengthening material generalization of relationship (1.8) is possible in accordance with the suggestions in [20].

Thus, taking account of (1.6) and (1.8) we obtain from (1.5) tensor-linear physical relationships

$$\dot{\boldsymbol{\varepsilon}} = \chi(\sigma_e)\,\varphi_*^q/(\varphi_* - \varphi)^q\left[\mathbf{a} \cdot\cdot\, \boldsymbol{\sigma}/\sigma_0 + \alpha\mathbf{b} + \gamma\left(\boldsymbol{\sigma} \cdot\cdot\, {}^{(6)}\mathbf{c} \cdot\cdot\, \boldsymbol{\sigma}/\sigma_2^2\right)\right] \tag{1.9}$$

and a kinetic equation

$$d\varphi/dt = v(\sigma_e)\,\varphi_*^q/(\varphi_* - \varphi)^q. \tag{1.10}$$

Function $\chi(\sigma_e) = v(\sigma_e)/\sigma_e$ in (1.9) is defined specifically on the basis of experimental reference data and it may be prescribed in the simplest form as a power relationship for σ_e^n, a hyperbolic sine rule $\sinh(\sigma_e/d)$, and with known reservations as an exponential representation $\exp(\sigma_e/p)$ (n, p, d are constants).

It is easy to establish that Eq. (1.9) stems from quite general relationships [27]

$$\dot{\boldsymbol{\varepsilon}} = \mathbf{H} + {}^{(4)}\mathbf{M} \cdot\cdot\, \boldsymbol{\sigma} + ({}^{(6)}\mathbf{L} \cdot\cdot\, \boldsymbol{\sigma}) \cdot\cdot\, \boldsymbol{\sigma}. \tag{1.11}$$

In our case tensor functions $\mathbf{H}$, ${}^{(4)}\mathbf{M}$, and ${}^{(6)}\mathbf{L}$ in (1.11) are found as

$$\mathbf{H} = \Psi\alpha\mathbf{b},\quad {}^{(4)}\mathbf{M} = \Psi^{(4)}\mathbf{a}/\sigma_0,\quad {}^{(6)}\mathbf{L} = \Psi\gamma^{(6)}\mathbf{c}/\sigma_2^2,$$

where $\Psi = v(\sigma_e)\,\varphi_*^q/[(\varphi_* - \varphi)^q\sigma_e]$.

2. We consider some particular forms for entries in Eqs. (1.9) and (1.10). First it is noted that as a result of symmetry for the stress tensor the tensors for the constants introduced are also symmetrical, ie.., they satisfy the conditions

$$b_{ij} = b_{ji},\ a_{ijkl} = a_{jikl} = a_{ijlk} = a_{klij},$$

$$c_{ijklmn} = c_{jiklmn} = c_{ijlkmn} = c_{ijklnm} = c_{klijmn} = c_{mnijkl} = c_{ijmnkl} = c_{mnklij} = c_{klmnij}.$$

Taking account of this invariant σ contains six independent constants, σ_1^2 contains 21, and σ_2^3 contains 56.

For orthotropic materials with coincidence of the coordinate axes with the principal directions of anisotropy relationships (1.9) take the form

$$\begin{gathered}
\dot{\varepsilon}_{11} = \Psi\big[(a_{1111}\sigma_{11} + a_{1122}\sigma_{22} + a_{1133}\sigma_{33})/\sigma_0 + \alpha b_{11} + \gamma\,(c_{111111}\sigma_{11}^2 + \\
+ c_{112222}\sigma_{22}^2 + c_{113333}\sigma_{33}^2 + 2c_{111122}\sigma_{11}\sigma_{22} + 2c_{111133}\sigma_{11}\sigma_{33} + 2c_{112233}\sigma_{22}\sigma_{33} + \\
+ 4c_{111212}\sigma_{12}^2 + 4c_{112323}\sigma_{23}^2 + 4c_{111313}\sigma_{13}^2)\,/\sigma_2^2\big], \\
\dot{\varepsilon}_{12} = \Psi\big[2a_{1212}\sigma_{12}/\sigma_0 + \gamma\,(4c_{121211}\sigma_{12}\sigma_{11} + 4c_{121222}\sigma_{12}\sigma_{22} + 4c_{121233}\sigma_{12}\sigma_{33} + 8c_{122313}\sigma_{23}\sigma_{13})/\sigma_2^2\big].
\end{gathered} \tag{2.1}$$

Here

$$\Psi = \chi(\sigma_e)\,\varphi_*^q/(\varphi_* - \varphi)^q;\ \sigma_1 = b_{11}\sigma_{11} + b_{22}\sigma_{22} + b_{33}\sigma_{33};$$

$$\sigma_0^2 = a_{1111}\sigma_{11}^2 + a_{2222}\sigma_{22}^2 + a_{3333}\sigma_{33}^2 + 2a_{1122}\sigma_{11}\sigma_{22} + 2a_{2233}\sigma_{22}\sigma_{33} + 2a_{1133}\sigma_{11}\sigma_{33} + 4a_{1212}\sigma_{12}^2 + 4a_{2323}\sigma_{23}^2 + 4a_{1313}\sigma_{13}^2;$$

$$\begin{gathered}
\sigma_2^3 = c_{111111}\sigma_{11}^3 + c_{222222}\sigma_{22}^3 + c_{333333}\sigma_{33}^3 + 3c_{111122}\sigma_{11}^2\sigma_{22} + 3c_{111133}\sigma_{11}^2\sigma_{33} + \\
+ 3c_{222211}\sigma_{22}^2\sigma_{11} + 3c_{222233}\sigma_{22}^2\sigma_{33} + 3c_{333311}\sigma_{33}^2\sigma_{11} + 3c_{333322}\sigma_{33}^2\sigma_{22} + \\
+ 6c_{112233}\sigma_{11}\sigma_{22}\sigma_{33} + 12c_{121211}\sigma_{12}^2\sigma_{11} + 12c_{121222}\sigma_{12}^2\sigma_{22} + 12c_{121233}\sigma_{12}^2\sigma_{33} + \\
+ 12c_{232311}\sigma_{23}^2\sigma_{11} + 12c_{232322}\sigma_{23}^2\sigma_{22} + 12c_{232333}\sigma_{23}^2\sigma_{33} + 12c_{131311}\sigma_{13}^2\sigma_{11} + \\
+ 12c_{131322}\sigma_{13}^2\sigma_{22} + 12c_{131333}\sigma_{13}^2\sigma_{33} + 48c_{122313}\sigma_{12}\sigma_{23}\sigma_{13};
\end{gathered}$$

and the rest of the physical relationships are obtained from (2.1) by circular transposition of indices 1, 2, and 3. It is emphasized that for the orthotropic material in question tensor b includes three independent constants, ${}^{(4)}a$ includes nine, and ${}^{(6)}c$ includes twenty.

In describing the creep of isotropic materials in Eqs. (2.1) it is necessary to place

$$\begin{gathered}
b_{ij} = B\delta_{ij},\ a_{ijkl} = A\delta_{ij}\delta_{kl} + (C/2)(\delta_{ik}\delta_{jl} + \delta_{li}\delta_{jk}),\ c_{ijklmn} = D\delta_{ij}\delta_{kl}\delta_{mn} + (K/6)(\delta_{ij}\delta_{km}\delta_{ln} + \delta_{ij}\delta_{kn}\delta_{lm} + \\
+ \delta_{kl}\delta_{im}\delta_{jn} + \delta_{kl}\delta_{in}\delta_{jm} + \delta_{mn}\delta_{ik}\delta_{jl} + \delta_{mn}\delta_{il}\delta_{jk}) + (E/8)(\delta_{ik}\delta_{jm}\delta_{ln} + \delta_{ik}\delta_{jn}\delta_{lm} + \delta_{il}\delta_{km}\delta_{jn} + \\
+ \delta_{il}\delta_{kn}\delta_{jm} + \delta_{im}\delta_{kj}\delta_{ln} + \delta_{im}\delta_{kn}\delta_{lj} + \delta_{in}\delta_{kj}\delta_{lm} + \delta_{in}\delta_{km}\delta_{lj})
\end{gathered}$$

(δ_{ij} is Kronecker symbol). Then fundamental relationships (2.1) are written out as

$$\dot{\varepsilon}_{ij} = \chi(\sigma_e)\,\varphi_*^q/(\varphi_* - \varphi)^q\,\{(AJ_1\delta_{ij} + C\sigma_{ij})/\sigma_0 + \alpha B\delta_{ij} + \gamma\,[DJ_1^2\delta_{ij} + E\sigma_{ik}\sigma_{kj} + K/3\,(J_2\delta_{ij} + 2J_1\sigma_{ij})]/\sigma_2^2\}, \tag{2.2}$$

where $\sigma_1 = BJ_1$; $\sigma_0^2 = AJ_1^2 + CJ_2$; $\sigma_2^3 = DJ_1^3 + KJ_1J_2 + EJ_3$; $J_1 = \sigma_{ij}\delta_{ij}$; $J_2 = \sigma_{ij}\sigma_{ij}$; $J_3 = \sigma_{ij}\sigma_{jk}\sigma_{ki}$. Thus, the equations suggested for isotropic creep use three invariants of the stress tensor (J_1, J_2, J_3) and six parameters (A, B, C, D, E, K) in expressions for σ_e.

We note that with practical application of physical relationships (2.1) and (2.2) relationships (1.3) and (1.10) are also involved which retain the same form of entry as in the case of more general equations (1.9).

3. We discuss possibilities for the approach in question on the example of creep for isotropic materials described by Eqs. (2.2). The conclusions and recommendations obtained here may then be used in analyzing relationships (1.9).

In order to determine the six parameters in (2.2) six test data are required. For this purpose we consider the following basic creep experiment up to failure in which a uniform stressed state is created in specimens of the test material:

uniaxial tension ($\sigma_{11} \neq 0$) establishing in the direction of the applied load the relationship

$$\dot{\varepsilon}_{11} = K_+ \sigma_{11}^n \varphi_*^q / (\varphi_* - \varphi)^q \tag{3.1}$$

and in the transverse direction $\dot{\varepsilon}_{22} = -Q\sigma_{11}^n \varphi_*^q/(\varphi_* - \varphi)^q$;

uniaxial compression ($\sigma_{11} \neq 0$) with which

$$\dot{\varepsilon}_{11} = -K_- |\sigma_{11}|^n \varphi_*^q/(\varphi_* - \varphi)^q; \tag{3.2}$$

pure torsion ($\sigma_{12} \neq 0$) with a specific change in angular velocity

$$2\dot{\varepsilon}_{12} = N\sigma_{12}^n \varphi_*^q/(\varphi_* - \varphi)^q \tag{3.3}$$

and axial $\dot{\varepsilon}_{11} = M\sigma_{12}^n \varphi_*^q/(\varphi_* - \varphi)^q$ strain;

loading by hydrostatic pressure ($\sigma_{11} = \sigma_{22} = \sigma_{33} = -|J_1|/3$) which establishes the rule $\dot{\varepsilon}_{11} = \dot{\varepsilon}_{22} = \dot{\varepsilon}_{33} = -P|\sigma_{11}|^n \varphi_*^q/(\varphi_* - \varphi)^q$; material constants K_+, K_-, Q, N, M, P, n, q, φ_* are assumed to be known.

Then by taking $\chi(\sigma_e) = \sigma_e^n$ in (2.2) and writing out for each of the stressed states considered above the corresponding equations which emerge from (2.2), we find after simple transformations the parameters in physical relationships:

$$\begin{gathered} C = N^{2r}/2;\ \alpha B = M/(\sqrt{2C})^n,\ A = X^2 - C, \\ 6\gamma^3 D = [\sqrt{9A + 3C} - 3\alpha B - (3P)^r]^3 - 3(T - \alpha B)^3 + 18(A/\sqrt{A + C} + \alpha B + QK_+^{-nr})(T - \alpha B)^2, \\ 2\gamma^3 K = 3(T - \alpha B)^3 - [\sqrt{9A + 3C} - 3\alpha B - (3P)^r]^3 - 24(A/\sqrt{A + C} + \alpha B + QK_+^{-nr})(T - \alpha B)^2, \\ \gamma^3 L = (T - \alpha B)^3 - \gamma^3 D - \gamma^3 K. \end{gathered} \tag{3.4}$$

Here $T = (K_+^r - K_-^r)/2$; $X = (K_+^r + K_-^r)/2$; $r = 1/(n + 1)$. Parameters in Eqs. (2.2) with other representations of $\chi(\sigma_e)$ are determined by a similar procedure.

We obtain partial physical relationships which emerge from (2.2) and which contain a smaller number of parameters. As before we assume that $\chi(\sigma_e) = \sigma_e^n$. If for example from the data of reference experiments the equality

$$T = MN^{-nr},\ \sqrt{9X^2 - 3N^{2r}} = 3T + (3P)^r, \tag{3.5}$$

is established, then from (3.4) we have $\gamma = 0$. Therefore $\sigma_e = \alpha\sigma_1 + \sigma_0$, i.e., in this case the equivalent stress does not contain the third stress tensor invariant. Then creep Eqs. (2.2) are written out in the form

$$\dot{\varepsilon}_{ij} = \chi(\sigma_e)\varphi_*^q/(\varphi_* - \varphi)^q \left(\frac{AJ_1\delta_{ij} + C\sigma_{ij}}{\sigma_0} + \alpha B\delta_{ij}\right), \tag{3.6}$$

and relationships (3.5) are conditions for using tensor-linear relationships (3.6). In the case of fulfilling the equalities

$$3T^3 - \left[\sqrt{9X^2 - 3N^{2r}} - (3P)^r\right]^3 = Y = M = 0, \tag{3.7}$$

where $Y = X = N^{2r}/(2X) + QK_+^{-nr}$, on the basis of Eqs. (3.4) we arrive at the requirements $\alpha B = D = K = 0$. Here relationships (2.2) degenerate into the equations

$$\dot{\varepsilon}_{ij} = \chi(\sigma_e)\varphi_*^q/(\varphi_* - \varphi)^q \left(\frac{AJ_1\delta_{ij} + C\sigma_{ij}}{\sigma_0} + \gamma\frac{L\sigma_{ik}\sigma_{kj}}{\sigma_1^2}\right). \tag{3.8}$$

The physical relationships (3.8) obtained as before for (2.2) are tensor-linear and use all three invariants of the stressed state, although they contain a smaller number of parameters compared with the original Eqs. (2.2). The possibilities for using relationships (3.8) are specified by equalities (3.7). If the relationships

$$\left[\sqrt{9X^2 - 3N^{2r}} - (3P)^r\right]^3 - 9T^3 \doteq T + 3Y = M = 0, \tag{3.9}$$

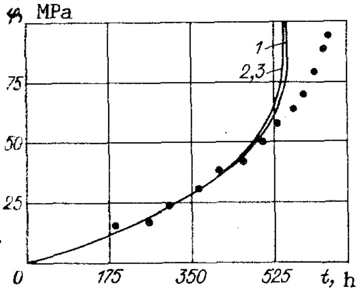

Fig. 1

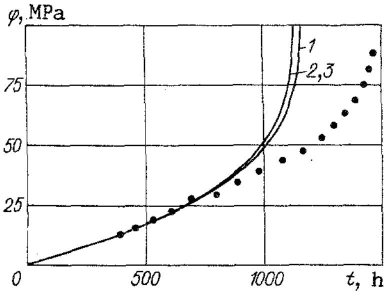

Fig. 2

are established by test data, then from (3.4) it follows that $\alpha B = D = L = 0$, and therefore Eqs. (2.2) take the form

$$\dot{\varepsilon}_{ij} = \chi(\sigma_e)\,\varphi_*^q/(\varphi_* - \varphi)^q \left[\frac{AJ_1\delta_{ij} + C\sigma_{ij}}{\sigma_0} + \gamma\frac{K(J_2\delta_{ij} + 2J_1\sigma_{ij})}{3\sigma_1^2}\right]. \qquad (3.10)$$

Here the conditions (3.9) will be recommendations for using relationships (3.10).

The three-parameter relationships (3.6), (3.8), and (3.10) considered are not the only equations which emerge from (2.2). For example, in fulfilling the equalities

$$K_+ = K_-,\ M = P = 0,\ N^{2r} = 3K_+^{2r} \qquad (3.11)$$

there follow from Eqs. (3.4) equalities $\alpha B = \gamma = 0$, $C = -3A$. Here physical relationships (2.2) degenerate into known equations [19]

$$\dot{\varepsilon}_{ij} = \sqrt{\tfrac{3}{2}C}\,\chi\left(\sqrt{\tfrac{2}{3}C}\,\sigma_i\right)\varphi_*^q/(\varphi_* - \varphi)^q\left(\sigma_{ij} - \tfrac{1}{3}J_1\delta_{ij}\right)$$

for materials which are not sensitive to the form of loading. Thus, relationships (2.2) exhibit sufficient generality and they include as special cases a number of fundamental equations.

Now we turn to experimental creep and stress-rupture data [7] for titanium alloy OT-4 with temperature T = 748 K. Unfortunately due to the lack in [7] of all of the results of the reference experiments formulated previously it is not possible to draw on Eqs. (2.2) and establish the validity of the particular procedural recommendations from (3.5), (3.7), (3.9). Therefore, we use the results of three tests (tension, compression, torsion) described by relationships (3.1)-(3.3). It was established in [7] that material constants are $K_+ = 13.3\cdot10^{-14}$ $MPa^{-n}\cdot h^{-1}$, $K_- = 7.5\cdot10^{-14}$ $MPa^{-n}\cdot h^{-1}$, $N = 27.7\cdot10^{-13}$ $MPa^{-n}\cdot h^{-1}$, $n = 4$, $q = 2$, $\varphi_* = 100$ MPa. By using these test data it is possible to determine parameters in physical relationships (3.6), (3.8), (3.10) taking $\chi(\sigma_e) = \sigma_e^n$.

We compare results calculated on the basis of relationships (3.6), (3.8), (3.10) with experimental data with a two-dimensional stressed state for the titanium alloy in question. Experiments were carried out [7] on thin-walled tubular specimens loaded by a tensile force and torsional moment. Presented in Figs. 1 and 2 is the change in specific energy dissipated φ with passage of time t in two experiments in tension with torsion: $\sigma_{11} = 194.9$ MPa, $\sigma_{12} = 46.6$ MPa and $\sigma_{11} = 156.3$ MPa, $\sigma_{12} = 52.1$ MPa, respectively [points are experimental data, lines 1-3 are the results of calculations relating to relationships (3.6), (3.8), (3.10)]. All of the calculations were carried out by integrating kinetic Eqs. (1.10) by the Kutt–Merson method with automatic selection of the step.

By considering the unique scatter of test data for creep and particularly marked in the third stage, the agreement of theoretical and experimental results may be considered satisfactory. Also noted is the similarity between calculated data obtained on the basis of physical relationships which describe in a different way the different resistance to tension and compression. In this situation it is correct to use, at least with a two-dimensional stressed state, simpler tensor-linear equations (3.6). A similar question for the three-dimensional stressed state remains open. However, it is an undoubted fact that nonfulfillment of even one of equalities (3.11) makes it impossible to use traditional creep equations which do not take account of the effect of the form of loading.

The theoretical assumptions, the basic reference experiments formulated, and the procedural recommendations developed in the present work may serve as a basis and direction for future theoretical and experimental studies in the field of creep for isotropic and anisotropic materials with a different resistance to tension and compression. Other approaches formulated in modern creep and stress-rupture theory for the materials in question, e.g., the energy version [28], are in no way repudiated.

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